Compton scattering on the charged pion and the process $\gamma\gamma \to \pi^0\pi^{0\star}$

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Abstract. The Compton scattering on a charged pion and the process $\gamma\gamma \to \pi^0\pi^0$ are studied using the dispersion relations. Unknown parameters of the S-wave $\pi\pi$ interaction and a sum and a difference of the π^0 meson polarizabilities are found from a fit to the experimental data for the $\gamma\gamma \to \pi^0\pi^0$ process. The found parameters of the $\pi\pi$ interaction are used for the calculation of the cross section of the elastic $\gamma\pi^\pm$ scattering. The analysis of the obtained results shows that the experimental data for the elastic $\gamma\pi^\pm$ scattering in the energy region up to 1 GeV together with the data for the $\gamma\gamma \to \pi^0\pi^0$ process could be used both for a determination of the pion polarizability values and for study of the S-wave $\pi\pi$ interaction.

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1 Introduction

At present the experiments at Mainz [1], CEBAF [2] and CERN [3] are planed to determine the cross section of the elastic $\gamma\pi^\pm$ scattering in the energy region $\sqrt{s} \leq 420$ MeV (where s is the square of the total energy in $\gamma\pi$ c.m.s.) and extract from it the pion polarizabilities. In the work [4] it has been shown that the corrections to the low energy expression for the cross section of the Compton scattering on $\pi^\pm$ meson in the energy region under consideration can be big. This could influence essentially the values of the polarizability obtained from these experiments using the low energy model. Therefore, it is necessary to find a correct theoretical expression for the cross section allowed to extract the pion polarizabilities with high precision in a wide enough energy region. Moreover, it is of interest to investigate what an information could be obtained from the analysis of experimental data for this process at higher energy.

The aim of the present work is an investigation of the elastic $\gamma\pi^\pm$ scattering in the energy region up to $\sqrt{s} \simeq 1$ GeV using the available experimental information about the process $\gamma\gamma \to \pi^0\pi^0$. For this purpose the dispersion relations (DRs) with the subtraction are constructed for the $\gamma\pi$ scattering amplitudes. Unknown parameters of the S-wave $\pi\pi$ interaction, represented as a broad Breit-Wigner resonance, are determined from fits to the experimental data [5] for the $\gamma\gamma \to \pi^0\pi^0$ process for which the same DRs are used.

An investigation of the process $\gamma\gamma \to \pi\pi$ at low energies was carried out in a number of works in the framework of the chiral perturbation theory (ChPT). In the case of the charged pions, the one-loop ChPT [6] describes quite well the experimental data [7] at low energy whereas such an approximation is in disagreement with the data for the $\gamma\gamma \to \pi^0\pi^0$ process. The agreement with these data was essentially improved by the two-loop calculation [8].

In the work [9] it was shown that the two-loop contributions induce only small changes in the cross section of the process $\gamma\gamma \to \pi^+\pi^-$. The author of this work studied also the Compton scattering on a charged pion and showed that the total cross section of this process in the energy region up to $\sqrt{s} = 350$ MeV is dominated by the Born term and one- and two-loop corrections are negligible.

The chiral perturbation approach is valid at low energies. To extend the energy region, the authors of the work [10] gave a description of different channels of $\gamma\gamma \to M\bar{M}$ reaction (where M were π, K, η mesons) using for the final state interaction of mesons the results obtained in the model [11] which combines coupled channel Lippmann-Schwinger equations with meson-meson potentials provided by the lowest order chiral Lagrangian. Their results are in good agreement with the experimental data for the $\gamma\gamma \to \pi^+\pi^-$ process up to $\sqrt{t} = 1.4$ GeV. However, in the case of the process $\gamma\gamma \to \pi^0\pi^0$ their results lie significantly higher than the experimental data [5] for $0.4\text{GeV} < \sqrt{t} < 0.6\text{GeV}$.

In work [12] the reaction $\gamma\gamma \to M\bar{M}$ was studied by using the master formula approach to QCD with three flavors. To go out beyond threshold region the authors used resonance saturation methods.

An analysis of the process $\gamma\gamma \to \pi\pi$ in the framework of DRs was performed early in the works [13–15]. These DRs were constructed for the S-wave amplitude and a correction for higher waves was realized by taking into account poles of the vector mesons.

We construct the DRs at fixed t with the subtraction for the invariant amplitudes and determine the subtraction functions through the DRs with the subtraction in cross channels and the difference and the sum of the pion polarizabilities. Such DRs allow to describe both the Compton scattering on the pion and the process $\gamma\gamma \to \pi\pi$ simultaneously and to estimate expected contributions of the pion polarizabilities in different kinematical regions of the process $\gamma + \pi^\pm \to \gamma + \pi^\pm$. The subtractions in the DRs provide a good enough convergence of underintegral expressions of these DRs and so increase the reliability of the calculations.

The content of this work is as follows. In Sect. 2 the DRs for the amplitudes of $\gamma\pi$ scattering are constructed. Determination of the parameters of the S-wave $\pi\pi$ interaction and the π^0 meson polarizabilities are in Sect. 3. In Sect. 4 the analysis of the Compton scattering on the charged pion is discussed and one suggests a method of extraction of the pion polarizability from the Compton scattering data in the energy region up to 1 GeV. The conclusions are given in Sect. 5.

2 Dispersion relations for the amplitudes of $\gamma\pi^\pm$ scattering

The Compton scattering on the pion is described by the following invariant variables

$$s = (p_1 + k_1)^2, \quad u = (p_1 - k_2)^2, \quad t = (k_1 - k_2)^2 \quad (1)$$

where $p_1(p_2)$ and $k_1(k_2)$ are the pion and photon initial (final) 4-momenta. The low energy expression for the cross section for Compton scattering on charged pion [4,16] is

$$\frac{d\sigma_{\gamma\pi}}{d\Omega} = \left(\frac{d\sigma_{\gamma\pi}}{d\Omega}\right)_B - \left(\frac{e^2}{4\pi}\right) \frac{\mu^3(s-\mu^2)^2}{4s^2[(s+\mu^2)+(s-\mu^2)z]}$$
$$\times \left\{(1-z)^2(\alpha-\beta)_{\pi^\pm} + \frac{s^2}{\mu^4}(1+z)^2(\alpha+\beta)_{\pi^\pm}\right\} \quad (2)$$

with $z = \cos\Theta_\gamma^{cm}$, the index B standing for the Born cross section, α and β are the electric and magnetic pion polarizabilities, respectively, and μ is the pion mass. The second term in this expression is equal to an interference of the Born amplitude with the pion polarizabilities.

In order to go out beyond the low-energy model let us construct DRs for the amplitudes of the elastic $\gamma\pi$ scattering. We will consider the helicity amplitudes M_{++} and M_{+-} which expressed through Prange's amplitudes [17] T_1 and T_2 as

$$M_{++} = -\frac{1}{2t}(T_1 + T_2),$$
$$M_{+-} = -\frac{T_1 - T_2}{2[(s-\mu^2)^2 + st]}. \quad (3)$$

These amplitudes have no kinematical singularities and zeros and define the cross section of the elastic $\gamma\pi$ scattering as the following:

$$\frac{d\sigma_{\gamma\pi}}{d\Omega} = \frac{1}{256\pi^2}\frac{(s-\mu^2)^4}{s^3}\left\{(1-z)^2|M_{++}|^2 + s^2(1+z)^2|M_{+-}|^2\right\}. \quad (4)$$

We construct for the amplitude M_{++} a DR at fixed t with one subtraction

$$ReM_{++}(s,t) = Re\overline{M}_{++}(s=\mu^2,t) + B_{++}$$
$$+\frac{(s-\mu^2)}{\pi}P\int_{4\mu^2}^{\infty}ds'\,ImM_{++}(s',t)\left[\frac{1}{(s'-s)(s'-\mu^2)}\right.$$
$$\left.-\frac{1}{(s'-u)(s'-\mu^2+t)}\right] \quad (5)$$

where B_{++} is the Born term equal to

$$B_{++} = \frac{2e^2\mu^2}{(s-\mu^2)(u-\mu^2)}, \quad (6)$$

and $Re\overline{M}$ is expressed through the difference of the pion polarizabilities as

$$Re\overline{M}_{++}(s=\mu^2, t=0) = 2\pi\mu(\alpha-\beta)_{\pi^\pm}. \quad (7)$$

Via the cross symmetry this DR is identical to a DR with two subtraction. The subtraction function $Re\overline{M}_{++}(s=\mu^2,t)$ is determined with help of the DR at fixed $s=\mu^2$ with one subtraction where the subtraction constant is expressed because of (7) through the difference $(\alpha-\beta)_{\pi^\pm}$:

$$Re\overline{M}_{++}(s=\mu^2, t)$$
$$= ReM_{++}(s=\mu^2,t) - B_{++}(s=\mu^2,t)$$
$$= 2\pi\mu(\alpha-\beta)_{\pi^\pm} + \frac{t}{\pi}\left\{P\int_{4\mu^2}^{\infty}\frac{ImM_{++}(t',s=\mu^2)\,dt'}{t'(t'-t)}\right.$$
$$\left.-P\int_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\,ds'}{(s'-\mu^2)(s'-\mu^2+t)}\right\}. \quad (8)$$

The DRs for the amplitude $M_{+-}(s,t)$ have the same expressions (5) and (8) with substitutions: $ImM_{++} \to ImM_{+-}$, $B_{++} \to B_{+-} = B_{++}/\mu^2$ and $2\pi\mu(\alpha-\beta)_{\pi^\pm} \to 2\pi/\mu(\alpha+\beta)_{\pi^\pm}$.

The dispersion sum rule (DSR) for the polarizability difference $(\alpha-\beta)$ can be obtained from the DR at fixed $u=\mu^2$ without subtraction for the amplitude M_{++} [18]:

$$(\alpha-\beta) = \frac{1}{2\pi^2\mu}\left\{\int_{4\mu^2}^{\infty}\frac{ImM_{++}(t',u=\mu^2)\,dt'}{t'}\right.$$
$$\left.+\int_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\,ds'}{s'-\mu^2}\right\}. \quad (9)$$

The DSR for the sum of the polarizabilities is

$$(\alpha + \beta) = \frac{\mu}{\pi^2} \int\limits_{4\mu^2}^{\infty} \frac{ImM_{+-}(s', t=0)\ ds'}{s' - \mu^2}$$

$$= \frac{1}{2\pi^2} \int\limits_{\frac{3}{2}\mu}^{\infty} \frac{\sigma_T(\nu)\ d\nu}{\nu^2} \qquad (10)$$

where σ_T is the total cross section of $\gamma\pi$ interaction, ν is the photon energy in lab. system.

The DRs and DSRs for the $\gamma\pi^{\pm}$ scattering are saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1260)$ and $a_2(1320)$ mesons in the s and u channels and σ, $f_0(980)$ and $f_2(1270)$ mesons in the t channel. In the case of the process $\gamma\gamma \to \pi^0\pi^0$ the DRs and DSRs are saturated by the contributions of the $\rho(770)$, $\omega(782)$ and $\phi(1020)$ mesons in the s and u channels and σ, $f_0(980)$ and $f_2(1270)$ mesons in the t channel.

3 Analysis of the process $\gamma\gamma \to \pi^0\pi^0$

The parameters of the ρ, ω, ϕ, b_1 and a_2 mesons are given by the Review of Particle Properties [19]. The parameters of the f_0 and f_2 mesons are taken from the work [5]. For the a_1 meson we take: $m_{a_1} = 1230$ MeV, $\Gamma_{a_1} = 450$ MeV and $\Gamma_{a_1 \to \gamma\pi} = 640$ keV. The parameters of the S-wave $\pi\pi$ interaction are not known. The main contribution to this interaction could be given by the σ meson.

The light scalar isoscalar meson (σ meson) with a mass approximately equal to twice the constituent mass of u and d quarks and with a big width is predicted to exist by a number of models for the dynamical breakdown of chiral symmetry. Such a particle is seen to occur, for example, within QCD adaptation of the Nambu-Jona-Lasinio model [20]. In the work [21] one has shown that the instanton vacuum structure predicted for QCD necessarily contains also such a meson in the 500–600 MeV range. In the last years search for the σ meson was carried out in a number of works [22–25]. However, experimental evidence for the σ meson is still both equivocal and controversial.

In the present work we will consider the σ meson as an effective description of the strong S-wave $\pi\pi$ interaction using the broad Breit-Wigner resonance expression. The parameters of such a σ meson are found from a fit to the experimental data [5] for the $\gamma\gamma \to \pi^0\pi^0$ process in the energy range of $\sqrt{t} = 270 \div 825$ MeV (where t is the square of total energy in $\gamma\gamma$ c.m.s.). For this reaction the Born term is equal to zero and the main contribution is determined by the S-wave of $\pi\pi$ interaction. So, this process gives a good possibility to investigate such a contribution. As the elastic $\gamma\pi$ scattering and the process $\gamma\gamma \to \pi\pi$ should be described by a common analytical function, we will use the same DRs for the description of these both processes.

There are five free parameters: the mass, the full width and the decay width into $\gamma\gamma$ of the σ meson and the sum
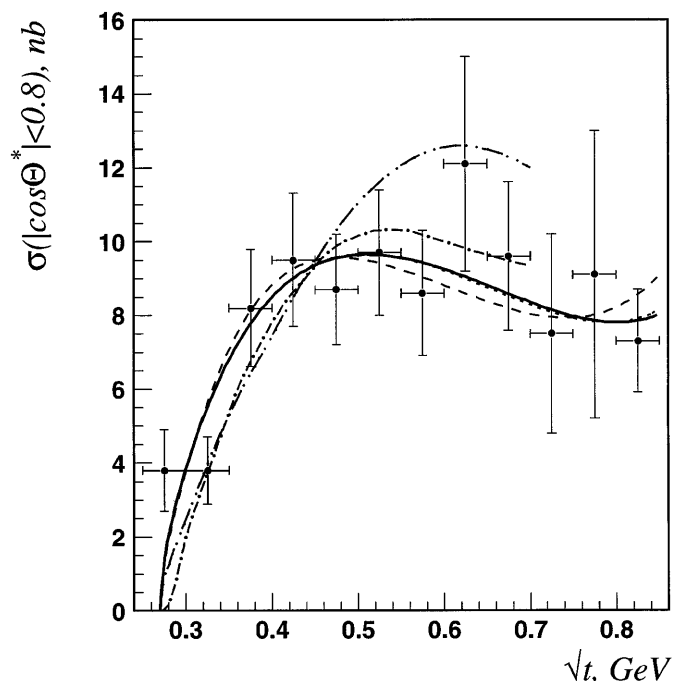

Fig. 1. The cross section for $\gamma\gamma \to \pi^0\pi^0$ process for $|\cos\theta^*| < 0.8$, where θ^* is the angle between the beam axis and one of the π^0 in the $\gamma\gamma$ center-of-mass system. The solid, dashed and dotted lines correspond to the "a", "b" and "c" variants of fitting. The dashed-dotted and dashed-double doted lines are the results of the work [15] and [8], respectively. Experimental data is taken from [5]

and the difference of the π^0 meson polarizabilities. Therefore, we consider the following variants of fitting:

a) For the first step we do a fit to the data [5] in the energy region from 270 MeV up to 2 GeV using all five parameters. This fit gives $(\alpha + \beta)_{\pi^0} = 0.98 \pm 0.03$ and $(\alpha - \beta)_{\pi^0} = -1.6 \pm 2.2$ (in units of 10^{-4}fm^3). Then we repeat the fit in the energy range of $\sqrt{t} = 270 \div 820$ MeV using the found values for the sum and the difference of the polarizabilities.

b) The sum and the difference of the polarizabilities are fixed according to two loops prediction of the ChPT [8]: $(\alpha + \beta)_{\pi^0} = 1.15 \pm 0.30$, $(\alpha - \beta)_{\pi^0} = -1.90 \pm 0.20$.

c) The values of these sum and difference are taken from the works [14, 26]: $(\alpha+\beta)_{\pi^0} = 1.00 \pm 0.05$, $(\alpha - \beta)_{\pi^0} = -0.6 \pm 1.8$.

Results of fitting in the energy region from $\sqrt{s} = 270$ MeV up to 825 MeV are presented in Fig. 1 by the solid line (the variant "a"), the dashed line (the variant "b") and the dotted one (the variant "c"). The dashed-dotted line shows the result of the work [15] of Donoghue and Holstein. The dashed-double dotted line is the result of the two-loop calculations in the framework of the ChPT [8]. As it is evident from this figure, our model essentially improved the agreement with the data [5].

The found parameters of the σ meson are listed in Table 1. The values of these parameters, in particular, the full width and the decay width into $\gamma\gamma$, depend strongly on

Table 1. Parameters of the σ meson obtained from different variants of the fit to the experimental data [5] of the process $\gamma\gamma \to \pi^0\pi^0$

	m_σ(MeV)	Γ_σ(MeV)	$\Gamma_{\sigma \to \gamma\gamma}$(keV)	χ^2	$(\alpha+\beta)_{\pi^0}$	$(\alpha-\beta)_{\pi^0}$
a	547 ± 45	1204 ± 362	0.62 ± 0.19	0.30	0.98 ± 0.03	-1.6 ± 2.2
b	471 ± 23	706 ± 164	0.33 ± 0.07	0.42	1.15 ± 0.30	-1.9 ± 0.2
c	584 ± 32	1378 ± 277	0.83 ± 0.16	0.31	1.00 ± 0.05	-0.6 ± 1.8

the value of the sum $(\alpha+\beta)_{\pi^0}$. The obtained values of the σ meson parameters indicate, probably, that in this case we should consider the σ meson as an effective account of the big contribution of the non-resonant S-wave $\pi\pi$ interaction rather than as a real particle.

The value of the sum of the π^0 meson polarizabilities, found in the variant "a" from the fit to the experimental data [5] on the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ in the energy region up to 2 GeV, practically coincides with the result obtained by Kaloshin et al. [26] from the analysis of angular distributions of the reaction under consideration in the region of the $f_2(1270)$ resonance. A very high sensitivity of the results of our calculations in this energy region to value of $(\alpha+\beta)_{\pi^0}$ led to the small value of its error.

The obtained value of the difference $(\alpha-\beta)_{\pi^0}$ agrees within the errors with the result of the work [14] where a simultaneous fit to data [5] and [7] was used. The big value of the error in this case is caused by of a weak dependence of our calculation results on the value of $(\alpha-\beta)_{\pi^0}$.

The found values of $(\alpha+\beta)_{\pi^0}$ and $(\alpha-\beta)_{\pi^0}$ are consistent within the error bars with the two-loop prediction of the ChPT [8].

Figure 2 demonstrates a description of the experimental data [5] of the process under investigation in the energy region up to $\sqrt{t} \approx 2$ GeV using the found values of the σ meson parameters and the corresponding values of the $(\alpha+\beta)_{\pi^0}$ and $(\alpha-\beta)_{\pi^0}$. The solid, dashed and dotted lines in this figure show results obtained for the "a", "b" and "c" variants, respectively. The result of calculations essentially depends on the value of $(\alpha+\beta)_{\pi^0}$. The dashed-dotted line in Fig. 2 corresponds to the values of the polarizabilities predicted in the quark confinement model [27]: $(\alpha+\beta)_{\pi^0} = 0.45$, $(\alpha-\beta)_{\pi^0} = 1.05$. This result is in complete disagreement with the experimental data in the region of the $f_2(1270)$ resonance.

The calculation of $(\alpha-\beta)_{\pi^0}$ with the help of the DSR (9) for the variant "a" gives:

$$(\alpha-\beta)_{\pi^0}^{DSR} = \overset{\rho}{-1.79} \overset{\omega}{-11.69} \overset{\phi}{-0.04} \overset{f_0}{+0.44} \overset{\sigma}{+10.07} = -3.01 \pm 2.06 \ .$$

The indicated error for this value is caused by the errors for the σ meson parameters only. The integration in this DSR was performed up to 25 GeV^2. The contribution into the error from a finite limit of the integration is about 10% in this case. This value of $(\alpha-\beta)_{\pi^0}$ does not conflict within the errors with the one used for the fit "a" and with the prediction of the ChPT [8].

The application of this DSR for calculation of the difference of the $\pi^\pm$ meson polarizabilities leads to the following value:
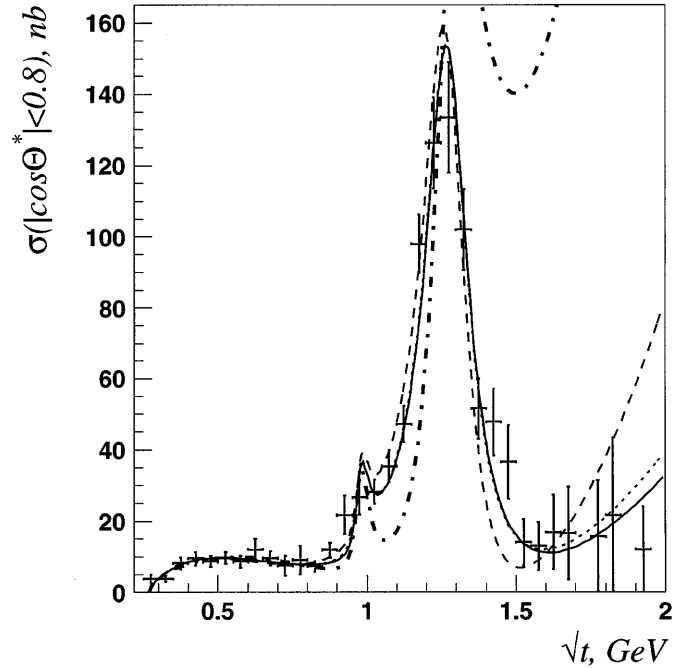

Fig. 2. The cross section for $\gamma\gamma \to \pi^0\pi^0$ process in the energy region up to 2 GeV. The solid, dashed and dotted lines correspond to the "a", "b" and "c" variants of fitting. The dashed-dotted line shows the result of calculations with the values of the polarizabilities from the work [27]. Experimental data is taken from [5]

$$(\alpha-\beta)_{\pi^\pm}^{DSR} = \overset{\rho}{-1.2} \overset{a_1}{+2.1} \overset{b_1}{+0.9} \overset{a_2}{-1.4} \overset{f_0}{+0.4} \overset{\sigma}{+9.5} = 10.3 \pm 1.9 \ .$$

This result differs from the prediction of the ChPT [28,9] (~ 5.6) and is close to the values obtained by DSRs [18,29] earlier. This difference might be caused by a slow convergence of the underintegral expressions in the DSR (9) and so an incorrect saturation of this DSR. Therefore, a more detail investigation of a saturation of this sum rule is necessary.

On the other hand, the DSR (10) has good enough convergence of the underintegral expression and gives the values of the sum polarizabilities [18,29] close to experimental ones.

It is worth noting that contrary to the DSR (9) the underintegral expressions in the DRs (5) and (8) converge, via of the subtractions, essentially more quickly and there is no such a problem with a saturation of these DRs. As a result, such DRs allow to obtain a good description of the experimental data [5] on the $\gamma\gamma \to \pi^0\pi^0$ process in the energy region up to 2 GeV (Fig. 2).

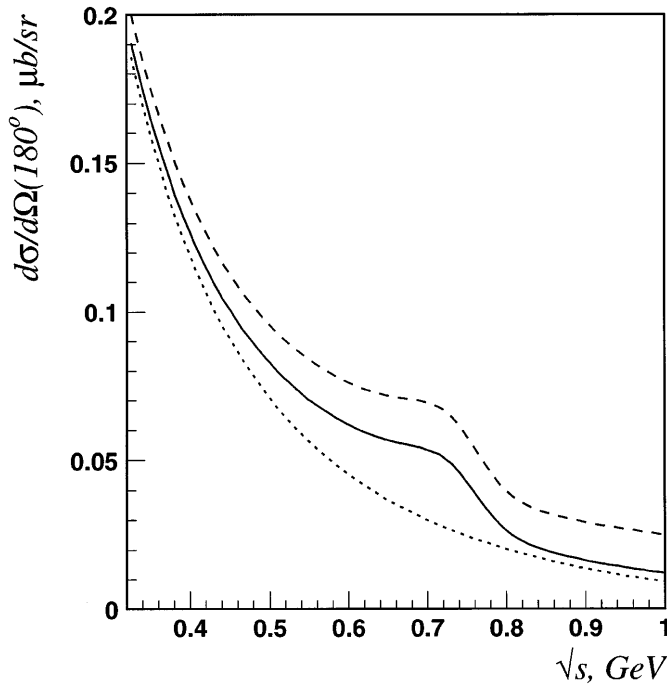**Fig. 3.** The back scattering cross section for $\gamma\pi^{\pm} \to \gamma\pi^{\pm}$ process. The solid line is the result of calculation for the "a" variant of the fitting at $(\alpha - \beta)_{\pi^{\pm}} = 5.6$, the dashed line corresponds to $(\alpha - \beta)_{\pi^{\pm}} = 0$, the dotted one is the contribution of the Born term $+ \pi^{\pm}$ meson polarizability
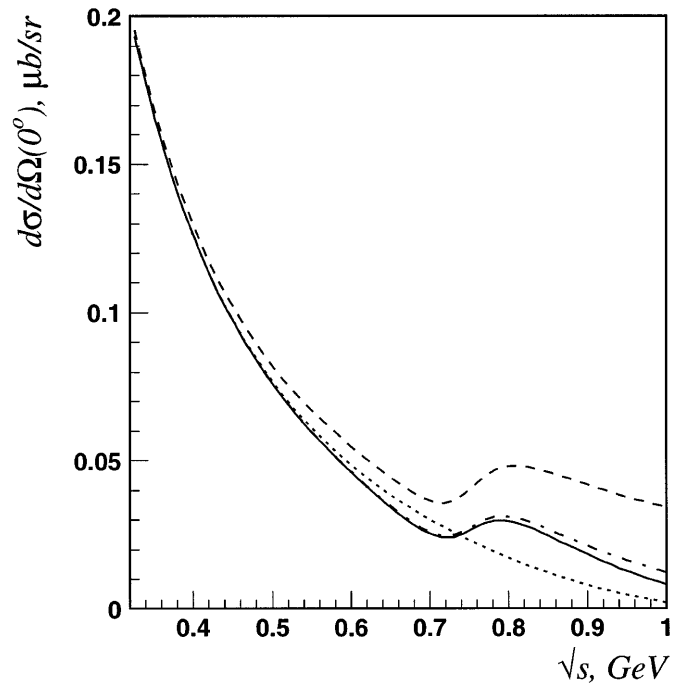

Fig. 4. The forward scattering cross section for $\gamma\pi^{\pm} \to \gamma\pi^{\pm}$ process at $(\alpha + \beta)_{\pi^{\pm}} = 0.22$ (solid line). The dashed line corresponds to $(\alpha + \beta)_{\pi^{\pm}} = 0$. The dotted line is the Born term + polarizability contribution, the dashed-dotted line corresponds to the contribution of the a_1, b_1 and a_2 mesons equal to zero

4 Analysis of Compton scattering on the $\pi^{\pm}$ meson

The calculation of the $\gamma\pi^{\pm}$ back scattering cross section with the help of the DRs (5) and (8) gave practically identical result for the "a", "b" and "c" variants of the fit to the data [5] for the process $\gamma\gamma \to \pi^0\pi^0$ (see Sect. 3). The maximum difference between these variants is realized at $\sqrt{s}$=1GeV and is smaller than 5%. The results of the calculation for the variant "a" at $(\alpha + \beta)_{\pi^{\pm}}$=0.22 and $(\alpha - \beta)_{\pi^{\pm}}$=5.6 are shown in Fig. 3 by the solid line. The dashed line corresponds to $(\alpha - \beta)_{\pi^{\pm}} = 0$. The dotted one is the contribution of the Born term + polarizabilities. The latter is identical to the low energy expression (2). It is evident from this figure that the correction to the low energy expression for the back scattering is important in a wide energy region. The analysis of the contribution of separate resonances showed, that the contribution of the ρ meson is essential only at the energy $\sqrt{s} > 500$ MeV, while the contribution of the σ meson is big in almost all considered energy region. The contribution of the b_1 and a_2 mesons is very small and even in the energy region of 850–1000 MeV it does not exceed 3.5%. The contribution of the a_1 meson becomes important at the energy higher than 750 MeV. This contribution increases from 8% up to 50% in the energy interval $750 - 1000$ MeV. Therefore, the data for the back Compton scattering on the charged pion at such energies can be used, in particular, to determine the decay widths of the a_1 meson.

For the $\gamma\pi^{\pm}$ forward scattering the σ meson does not give the contribution. In this case the cross section is determined by the contribution of the Born term, the sum $(\alpha + \beta)_{\pi^{\pm}}$ and the ρ, a_1, b_1 and a_2 mesons. The results of calculation of the forward scattering cross section are given in Fig. 4 where the solid line corresponds to account of all contributions with $(\alpha + \beta)_{\pi^{\pm}}$=0.22, the dashed and dashed-dotted lines show the results of the similar calculation but with $(\alpha + \beta)_{\pi^{\pm}} = 0$ and with the contribution of the a_1, b_1 and a_2 mesons equal to zero, respectively. The dotted line is contribution of the Born+$(\alpha + \beta)_{\pi^{\pm}}$. This figure demonstrates the big contribution of the sum $(\alpha + \beta)_{\pi^{\pm}}$ in the energy region near 1GeV which increases with the energy in both relative and absolute values. Corrections to the low energy expression (2) are smaller than 1% at $\sqrt{s} < 500$ MeV and become essential at higher energies. The main contribution into these corrections is given by the ρ meson. The contributions of the a_1, b_1 and a_2 mesons are small at $\sqrt{s} < 850$ MeV. They grow from 2–4% at $\sqrt{s} = 850$ MeV up to 12–16% at $\sqrt{s} = 1000$ MeV.

The Fig. 5 shows the result of calculation of the total cross section for the Compton scattering on the $\pi^{\pm}$ meson (solid line). The dashed line is the cross section at $\alpha = \beta = 0$ and dotted one is the contribution of the Born+polarizabilities. This figure demonstrates the possibility of the use of the data on the total cross section of the elastic $\gamma\pi^{\pm}$ scattering at the energy region higher than ~ 600 MeV for the extraction of the pion polarizability. In the energy region up to 700 MeV the total cross section is
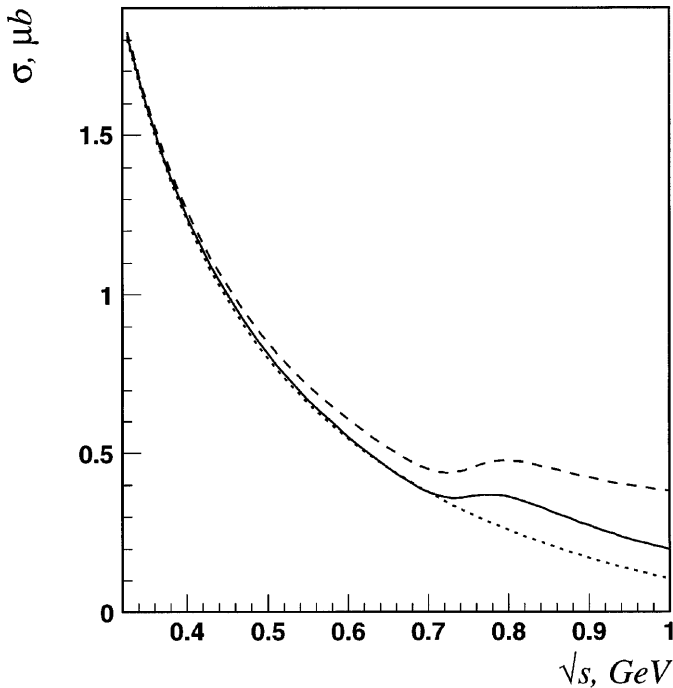
Fig. 5. The total cross section for the elastic $\gamma\pi^\pm$ scattering at $(\alpha - \beta)_{\pi^\pm} = 5.6$ and $(\alpha + \beta)_{\pi^\pm} = 0.22$ (solid line). The dashed line is the cross section at $\alpha = \beta = 0$ and dotted one is the contribution of the Born+polarizabilities
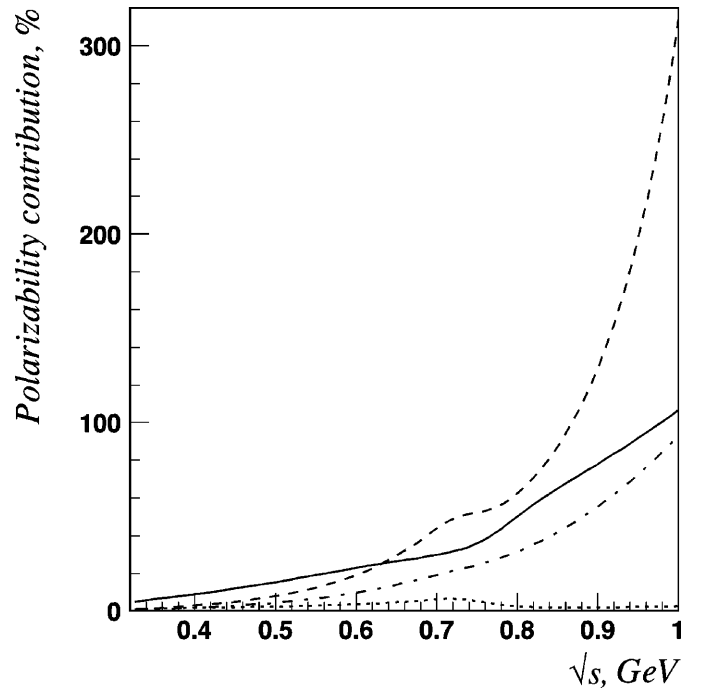
Fig. 6. The relative contributions of $(\alpha - \beta)_{\pi^\pm} = 5.6$ (solid line) and $(\alpha + \beta)_{\pi^\pm} = 0.22$ (dashed line) into the cross section for the back and forward scattering, respectively. The dotted and dashed-dotted lines correspond to the contribution of $(\alpha - \beta)_{\pi^\pm}$ and $(\alpha + \beta)_{\pi^\pm}$ into the total cross section of the elastic $\gamma\pi^\pm$ scattering, respectively

dominated by the low energy approach. In particular, at $\sqrt{s} = 350$ MeV the correction to this approach is equal to about 0.6% what is in agreement with the two-loop ChPT calculation [9].

The relative contributions of $(\alpha - \beta)_{\pi^\pm} = 5.6$ and $(\alpha + \beta)_{\pi^\pm} = 0.22$ into the back and forward cross sections, respectively, and their contribution into the total cross section as a function of the energy $\sqrt{s}$ are shown in Fig. 6. As follows from this figure the relative contributions of the $(\alpha - \beta)_{\pi^\pm}$ and $(\alpha + \beta)_{\pi^\pm}$ into the back and forward cross sections grow with the energy and in the region of 1GeV exceed 100% and 300%, respectively. The contribution of the polarizability into the total cross section is determined mainly by the sum $(\alpha + \beta)_{\pi^\pm}$ which also reaches 100% in this energy region. These results permit to determine the pion polarizabilities with good enough accuracy from the experimental data for the elastic $\gamma\pi^\pm$ scattering in the energy region under consideration. As follows from above-made analysis the most model independent result can be obtained in the energy region up to 750–850 MeV.

To obtain an expression for the extraction of the reliable value of $(\alpha - \beta)_{\pi^\pm}$ from the experimental data for the $\gamma\pi^\pm$ back scattering let us present the differential cross section of this process as the following:

$$\frac{d\sigma_{\gamma\pi}}{d\Omega} = (\alpha - \beta)^2_{\pi^\pm} A_1 + (\alpha - \beta)_{\pi^\pm} A_2 + A_3 \qquad (11)$$

where

$$A_1 = \frac{1}{64} \frac{(s - \mu^2)^4}{s^3} \mu^2 (1 - z)^2,$$

$$A_2 = \frac{1}{64\pi} \frac{(s - \mu^2)^4}{s^3} \mu (1 - z)^2 \tilde{M}_{++},$$

$$A_3 = \frac{1}{256\pi^2} \frac{(s - \mu^2)^4}{s^3} \left\{ (1 - z)^2 \left[\tilde{M}^2_{++} + Im M^2_{++} \right] + s^2 (1 + z)^2 |M_{+-}|^2 \right\}, \qquad (12)$$

$$\tilde{M}_{++} = Re M_{++} - 2\pi\mu(\alpha - \beta)_{\pi^\pm}.$$

The amplitudes $\tilde{M}_{++}$ and M_{+-} can be calculated using the upper constructed DRs.

As the first term in (10) is very small in a wide enough range of the energy, it can be neglected and we have

$$(\alpha - \beta)_{\pi^\pm} = \frac{\dfrac{d\sigma^{exp}_{\gamma\pi}}{d\Omega} - A_3}{A_2}. \qquad (13)$$

Here $d\sigma^{exp}_{\gamma\pi}/d\Omega$ is the experimental value of the $\gamma\pi^\pm$ scattering cross section. However, in the region of 1GeV the first term contribution in (11) could be about 10%. Therefore, to find the value of $(\alpha - \beta)_{\pi^\pm}$ with high enough accuracy we should use in this energy region the full equation (11). In the case of the experiment [1] the expressions (11) and (12) must be integrated over z from -1 up to -0.766.

For the forward scattering we write the cross section as

$$\frac{d\sigma_{\gamma\pi}}{d\Omega} = (\alpha + \beta)^2_{\pi^\pm} B_1 + (\alpha + \beta)_{\pi^\pm} B_2 + B_3 \qquad (14)$$

where

$$B_1 = \frac{1}{64} \frac{(s-\mu^2)^4}{s\mu^2}(1+z)^2,$$

$$B_2 = \frac{1}{64\pi} \frac{(s-\mu^2)^4}{s\mu}(1+z)^2 \tilde{M}_{+-},$$

$$B_3 = \frac{1}{256\pi^2} \frac{(s-\mu^2)^4}{s^3} \left\{ (1-z)^2 |M_{++}|^2 \right.$$
$$\left. + s^2(1+z)^2 \left[\tilde{M}_{+-}^2 + ImM_{+-}^2 \right] \right\}, \quad (15)$$

$$\tilde{M}_{+-} = ReM_{+-} - \frac{2\pi}{\mu}(\alpha + \beta)_{\pi^\pm}.$$

Neglecting the first term in (13) we have

$$(\alpha + \beta)_{\pi^\pm} = \frac{\dfrac{d\sigma_{\gamma\pi}^{exp}}{d\Omega} - B_3}{B_2}. \quad (16)$$

5 Conclusions

• The DRs at fixed t with one subtraction have been constructed for the invariant helicity amplitudes of the $\gamma\pi$ scattering. Using these DRs for the description of the process $\gamma\gamma \to \pi^0\pi^0$ we have obtained from the fit to the experimental data [5] for this process the sum and difference of the π^0 meson polarizabilities: $((\alpha + \beta)_{\pi^0} = 0.98 \pm 0.03$, $(\alpha - \beta)_{\pi^0} = -1.6 \pm 2.2)$, and found the parameters of the S-wave $\pi\pi$ interaction represented by the broad Breit-Wigner resonance formula.

• The analysis of the calculation of the cross section of elastic $\gamma\pi^\pm$ scattering showed that the S-wave $\pi\pi$ interaction gives the significant contribution into the cross section of the $\gamma\pi$ back scattering. However, the value of this contribution does not depend practically on the variant of fitting of the process $\gamma\gamma \to \pi^0\pi^0$ with the aim to find parameters of this interaction.

• One suggested the method of the extraction of the pion polarizabilities from the Compton scattering on the pion data in a wide energy region which took into account corrections to the low energy expression.

• One found kinematical regions where the corrections to the low energy expression either were very small or good calculated.

• The contributions of $(\alpha - \beta)_{\pi^\pm}$ (into the back scattering) and $(\alpha + \beta)_{\pi^\pm}$ (into the forward scattering and into the total cross section) grow with the energy and exceed 100% in the region of 1 GeV. The most model independent result of the pion polarizabilities extraction from the Compton scattering on the charged pion can be obtained in the energy region up to 750–850 MeV.

• The experimental data for the elastic $\gamma\pi^\pm$ scattering in the energy region up to ~ 1 GeV together with the $\gamma\gamma \to \pi^0\pi^0$ data could be used both for a determination of the pion polarizability values and for study of the S-wave $\pi\pi$ interaction.

Appendix

The contributions of the vector and axial-vector mesons (ρ, ω, ϕ and a_1) are calculated with the help of the expression

$$ImM_{++}^{(V)}(s,t) = \mp s \, ImM_{+-}^{(V)}(s,t)$$
$$= \mp 4g_V^2 s \frac{\Gamma_0}{(m_V^2 - s)^2 + \Gamma_0^2} \quad (A1)$$

where m_V is the meson mass, the sign "+" corresponds to the contribution of the a_1 meson and

$$g_V^2 = 6\pi \sqrt{\frac{m_V^2}{s}} \left(\frac{m_V}{m_V^2 - \mu^2} \right)^3 \Gamma_{V\to\gamma\pi},$$

$$\Gamma_0 = \left(\frac{s - 4\mu^2}{m_V^2 - 4\mu^2} \right)^{\frac{3}{2}} \Gamma_V m_V. \quad (A2)$$

Here Γ_V and $\Gamma_{V\to\gamma\pi}$ are the full width and the decay width into $\gamma\pi$ of these mesons, respectively.

The b_1 and a_2 mesons give relatively small contribution in the energy region under consideration. Therefore, we calculate it using a narrow width approximation.

$$ImM_{++}^{(b_1)}(s,t) = s \, ImM_{+-}^{(b_1)}(s,t) = 4g_b^2 \pi s \delta(s-m_b^2), \quad (A3)$$

$$ImM_{++}^{(a_2)}(s,t) = -\frac{1}{2}g_a^2 \pi \left[s^2 - s(4\mu^2 - t) + \mu^4 \right.$$
$$\left. - \frac{s(s-\mu^2)^2}{2m_a^2} \right] \delta(s-m_a^2), \quad (A4)$$

$$ImM_{+-}^{(a_2)}(s,t) = -\frac{1}{2}g_a^2 \pi \left[\mu^2 - t - \frac{(s-\mu^2)^2}{4m_a^2} \right] \delta(s-m_a^2)$$
$$(A5)$$

where

$$g_b^2 = 6\pi \left(\frac{m_b}{m_b^2 - \mu^2} \right)^3 \Gamma_{b_1 \to \gamma\pi^\pm},$$

$$g_a^2 = 160\pi \left(\frac{m_a}{m_a^2 - \mu^2} \right)^5 \Gamma_{a_2 \to \gamma\pi^\pm}.$$

For calculation of the contribution of the σ, f_0 and f_2 mesons we use the following expressions:

$$ImM_{++}^\sigma(t,s) = \frac{g_\sigma \Gamma_{0\sigma}}{(m_\sigma^2 - t)^2 + \Gamma_{0\sigma}^2},$$

$$ImM_{++}^{f_0}(t,s) = \frac{g_{f_0} \Gamma_{0f_0}}{(m_{f_0}^2 - t)^2 + \Gamma_{0f_0}^2}, \quad (A6)$$

$$ImM_{+-}^{f_2}(t,s) = \frac{g_{f_2}\Gamma_{0f_2}}{(m_{f_2}^2 - t)^2 + \Gamma_{0f_2}^2} \qquad (A7)$$

where

$$g_\sigma = 8\pi \frac{m_\sigma + \sqrt{t}}{\sqrt{t}} \left(\frac{\frac{2}{3}\Gamma_{\sigma\to\pi\pi}\Gamma_{\sigma\to\gamma\gamma}}{m_\sigma\sqrt{m_\sigma^2 - 4\mu^2}} \right)^{\frac{1}{2}},$$

$$g_{f_0} = 16\pi \left(\frac{\frac{2}{3}\Gamma_{f_0\to\pi\pi}\Gamma_{f_0\to\gamma\gamma}}{m_{f_0}\sqrt{m_{f_0}^2 - 4\mu^2}} \right)^{\frac{1}{2}},$$

$$\Gamma_{0\sigma} = \frac{\Gamma_\sigma}{2}(\sqrt{t} + m_\sigma)\left(\frac{t - 4\mu^2}{m_\sigma^2 - 4\mu^2} \right)^{\frac{1}{2}},$$

$$\Gamma_{0f_0} = \Gamma_{f_0} m_{f_0} \left(\frac{t - 4\mu^2}{m_{f_0}^2 - 4\mu^2} \right)^{\frac{1}{2}}, \qquad (A8)$$

$$g_{f_2} = 160\pi \frac{m_{f_2}^{3/2}}{t(m_{f_2}^2 - 4\mu^2)^{\frac{5}{4}}}\sqrt{\frac{D_2(m_{f_2}^2)}{D_2(t)}}\ \Gamma_{f_2\to\pi\pi}\Gamma_{f_2\to\gamma\gamma},$$

$$\Gamma_{0f_2} = \Gamma_{f_2}\frac{m_{f_2}}{\sqrt{t}} \left(\frac{t - 4\mu^2}{m_{f_2}^2 - 4\mu^2} \right)^{\frac{5}{2}} \frac{D_2(m_{f_2}^2)}{D_2(t)} \qquad (A9)$$

The decay form factor D_2 is given according to [5]

$$D_2(t) = 9 + 3(qr)^2 + (qr)^4, \qquad q^2 = \frac{1}{4}(t - 4\mu^2) \quad (A10)$$

and the effective interaction radius r is assumed to be 1fm. The factor $(m_\sigma + \sqrt{t})$ at the relations for g_σ and $\Gamma_{0\sigma}$ is introduced to get a more correct expression for a broad Breit-Wigner resonance.

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